

Connectivity Analysis of Dendrimer Nanostars: Multiplicative Product and Sum Indices Mikhail Ivanov¹, Tatiana Petrova²

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ABSTRACT

In Chemical Graph Theory, the connectivity indices are applied to measure the chemical characteristics of compounds. In this paper, we compute the multiplicative product connectivity index and the multiplicative sum connectivity index of three infinite families $NS_1[n]$, $NS_2[n]$, $NS_3[n]$ dendrimer nanostars.

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KEYWORDS: multiplicative product connectivity index, multiplicative sum connectivity index, dendrimer nanostar.

I. INTRODUCTION

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v . We refer to [1] for undefined term and notation.

A molecular graph or a chemical graph is a finite, simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecular is called topological index of that graph. There are several topological indices that have some applications in chemistry in *QSPR/QSAR* study [2].

Motivated by the definition of the product connectivity index and its wide applications, Kulli [3] introduced the multiplicative product connectivity index and multiplicative sum connectivity index of a molecular graph as follows:

The multiplicative product connectivity index of a graph G is defined as

$$\chi_{II}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$

The multiplicative sum connectivity index of a graph G is defined as

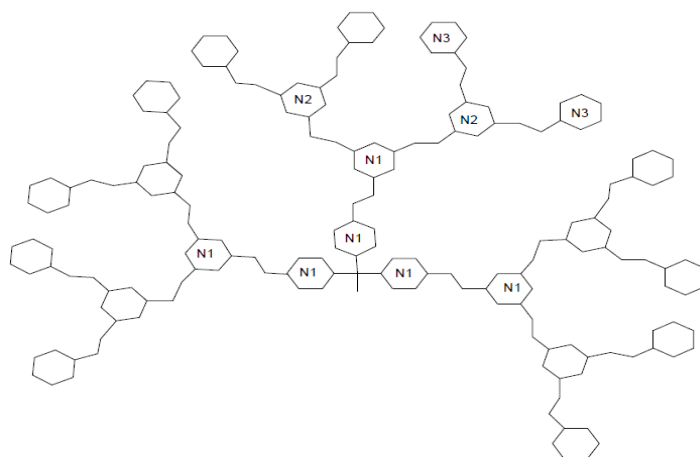
$$\chi_{III}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}.$$

Recently many multiplicative indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Also some connectivity indices were studied, for example, in [19, 20, 21, 22, 23, 24, 25].

In this paper, we compute the multiplicative product connectivity index and multiplicative sum connectivity index for three infinite classes $NS_1[n]$, $NS_2[n]$ and $NS_3[n]$ dendrimer nanostars. For more information about these dendrimer nanostars see [26, 27].

II. RESULTS FOR $NS_1[n]$ DENDRIMER NANOSTARS

We consider the first class of dendrimer nanostars. This family of dendrimer nanostars is symbolized by $NS_1[n]$, where n is steps of growth in this type of dendrimer nanostars. The graph of $NS_1[3]$ dendrimer nanostar is presented in Figure 1.

Figure 1. The graph of $NS_1[3]$

In the following theorem, we compute the multiplicative product connectivity index of $NS_1[n]$ dendrimer nanostars.

Theorem 1. The multiplicative product connectivity index of $NS_1[n]$ dendrimer nanostar is

$$\chi_{II}(NS_1[n]) = \left(\frac{1}{2\sqrt{3}}\right)^3 \times \left(\frac{1}{2}\right)^{9 \times 2^n + 4} \times \left(\frac{1}{6}\right)^{9 \times 2^n - 6}.$$

Proof: Let G be the graph of $NS_1[n]$ dendrimer nanostar. By calculation, we obtain that G has $27 \times 2^n - 5$ edges. We can see easily that the vertices of $NS_1[n]$ are of degree 1, 2, 3 or 4, see Figure 1. Also by calculation, we obtain that G has four types of edges based on the degree of end vertices of each edge as given in Table 1.

Table 1. Edge partition of $NS_1[n]$

$d_G(u), d_G(v) uv \in E(G)$	(1, 4)	(2, 2)	(2, 3)	(3, 4)
Number of edges	1	$9 \times 2^n + 3$	$18 \times 2^n - 12$	3

$$\text{We have } \chi_{II}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

By using Table 1, we have

$$\begin{aligned} \chi_{II}(NS_1[n]) &= \left(\frac{1}{\sqrt{1 \times 4}}\right)^1 \times \left(\frac{1}{\sqrt{2 \times 2}}\right)^{9 \times 2^n + 3} \times \left(\frac{1}{\sqrt{2 \times 3}}\right)^{18 \times 2^n - 12} \times \left(\frac{1}{\sqrt{3 \times 4}}\right)^3 \\ &= \left(\frac{1}{2\sqrt{3}}\right)^3 \times \left(\frac{1}{2}\right)^{9 \times 2^n + 4} \times \left(\frac{1}{6}\right)^{9 \times 2^n - 6}. \end{aligned}$$

In the next theorem, we compute the multiplicative sum connectivity index of $NS_1[n]$ dendrimer nanostars.

Theorem 2. The multiplicative sum connectivity index of $NS_1[n]$ dendrimer nanostar is

$$X_{II}(NS_1[n]) = \left(\frac{1}{\sqrt{7}}\right)^3 \times \left(\frac{1}{2}\right)^{9 \times 2^n + 3} \times \left(\frac{1}{\sqrt{5}}\right)^{18 \times 2^n - 11}.$$

$$\text{Proof: We have } X_{II}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}.$$

By using Table 1, we have

$$\begin{aligned} X_{II}(NS_1[n]) &= \left(\frac{1}{\sqrt{1+4}}\right)^1 \times \left(\frac{1}{\sqrt{2+2}}\right)^{9 \times 2^n + 3} \times \left(\frac{1}{\sqrt{2+3}}\right)^{18 \times 2^n - 12} \times \left(\frac{1}{\sqrt{3+4}}\right)^3 \\ &= \left(\frac{1}{\sqrt{7}}\right)^3 \times \left(\frac{1}{2}\right)^{9 \times 2^n + 3} \times \left(\frac{1}{\sqrt{5}}\right)^{18 \times 2^n - 11}. \end{aligned}$$

III. RESULTS FOR $NS_2[n]$ DENDRIMER NANOSTARS

We consider the second class of dendrimer nanostars. This family of dendrimer nanostars is symbolized by $NS_2[n]$, where n is steps of growth in this type of dendrimer nanostars. The graph of $NS_2[2]$ dendrimer nanostar is shown in Figure 2.

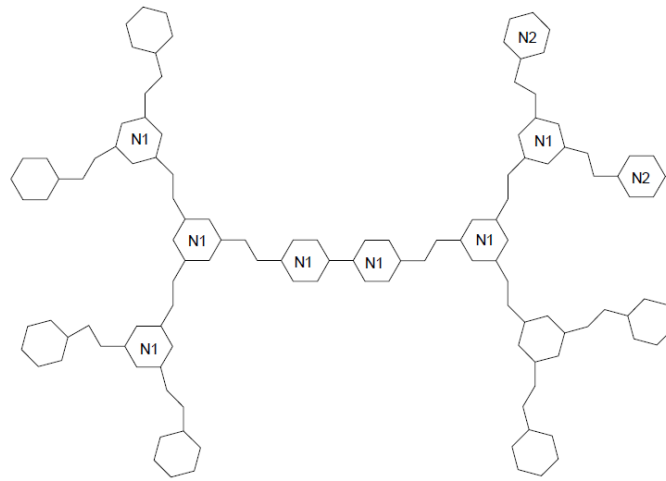


Figure 2 The graph of $NS_2[2]$

In the following theorem, we compute the multiplicative product connectivity index of $NS_2[n]$ dendrimer nanostar.

Theorem 3. The multiplicative product connectivity index of $NS_2[n]$ dendrimer nanostar is

$$\chi_{II}(NS_2[n]) = \left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right)^{12 \times 2^n + 2} \times \left(\frac{1}{6}\right)^{12 \times 2^n - 4}.$$

Proof: Let H be the graph of $NS_2[n]$ dendrimer nanostar. By calculation, we obtain that H has $36 \times 2^n - 5$ edges. One can see easily that the vertices of $NS_2[n]$ are of degree 2 or 3, see Figure 2. Also by calculation, we obtain that H has three types of edges based on the degree of end vertices of each edge as given in Table 2.

Table 2. Edge partition of $NS_2[n]$

$d_H(u), d_H(v) \mid uv \in E(H)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	$12 \times 2^n + 2$	$24 \times 2^n - 8$	1

We have $\chi_{II}(H) = \prod_{uv \in E(H)} \frac{1}{\sqrt{d_H(u)d_H(v)}}$.

By using Table 1, we have

$$\begin{aligned} \chi_{II}(NS_2[n]) &= \left(\frac{1}{\sqrt{2 \times 2}}\right)^{12 \times 2^n + 2} \times \left(\frac{1}{\sqrt{2 \times 3}}\right)^{24 \times 2^n - 8} \times \left(\frac{1}{\sqrt{3 \times 3}}\right)^1 \\ &= \left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right)^{12 \times 2^n + 2} \times \left(\frac{1}{6}\right)^{12 \times 2^n - 4}. \end{aligned}$$

In the next theorem, we compute the multiplicative sum connectivity index of $NS_2[n]$ dendrimer nanostar.

Theorem 4. The multiplicative sum connectivity index of $NS_2[n]$ dendrimer nanostar is

$$X_{II}(NS_2[n]) = \left(\frac{1}{\sqrt{6}}\right) \times \left(\frac{1}{2}\right)^{12 \times 2^n + 2} \times \left(\frac{1}{\sqrt{5}}\right)^{24 \times 2^n - 8}.$$

Proof: We have $X_{II}(H) = \prod_{uv \in E(H)} \frac{1}{\sqrt{d_H(u) + d_H(v)}}$

By using Table 1, we have

$$\begin{aligned} \chi II(NS_2[n]) &= \left(\frac{1}{\sqrt{2+2}}\right)^{12 \times 2^n + 2} \times \left(\frac{1}{\sqrt{2+3}}\right)^{24 \times 2^n - 8} \times \left(\frac{1}{\sqrt{3+3}}\right)^1 \\ &= \left(\frac{1}{\sqrt{6}}\right) \times \left(\frac{1}{2}\right)^{12 \times 2^n + 2} \times \left(\frac{1}{\sqrt{5}}\right)^{24 \times 2^n - 8}. \end{aligned}$$

IV. RESULTS FOR $NS_3[n]$ DENDRIMER NANOSTARS

We consider the third class of dendrimer nanostars. This family of dendrimer nanostars is symbolized by $NS_3[n]$, where n is steps of growth in this type of dendrimer nanostar. The graph of $NS_3[n]$ dendrimer nanostar is presented in Figure 3.

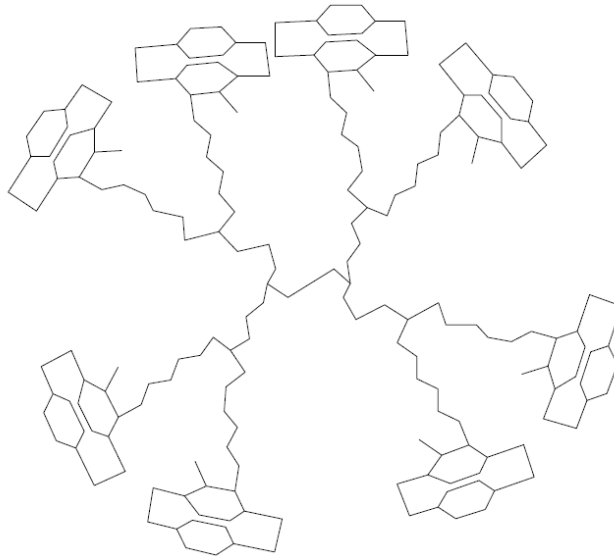


Figure 3. The graph of $NS_3[n]$

In the following theorem, we compute the multiplicative product connectivity index of $NS_3[n]$ dendrimer nanostars.

Theorem 5. The multiplicative product connectivity index of $NS_3[n]$ dendrimer nanostar is

$$\chi II(NS_3[n]) = \left(\frac{1}{3}\right)^{7 \times 2^n} \times \left(\frac{1}{2}\right)^{22 \times 2^n - 7} \times \left(\frac{1}{6}\right)^{14 \times 2^n - 3}.$$

Proof: Let G be the graph of $NS_3[n]$ dendrimer nanostar. By calculation, we obtain that G has $58 \times 2^n - 13$ edges. We can see easily that the vertices of $NS_3[n]$ are of degree 1, 2 or 3, see Figure 3. Also by calculation, we obtain that G has four types of edges based on the degree of end vertices of each edge as given in Table 3.

Table 3. Edge partition of $NS_3[n]$

$d_G(u), d_G(v) \mid uv \in E(G)$	(1, 3)	(2, 2)	(2, 3)	(3, 3)
Number of edges	2^{n+1}	$22 \times 2^n - 7$	$28 \times 2^n - 6$	6×2^n

We have
$$\chi II(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

By using Table 3, we have

$$\begin{aligned} \chi II(NS_3[n]) &= \left(\frac{1}{\sqrt{1 \times 3}}\right)^{2^{n+1}} \times \left(\frac{1}{\sqrt{2 \times 2}}\right)^{22 \times 2^n - 7} \times \left(\frac{1}{\sqrt{2 \times 3}}\right)^{28 \times 2^n - 6} \times \left(\frac{1}{\sqrt{3 \times 3}}\right)^{6 \times 2^n} \\ &= \left(\frac{1}{3}\right)^{7 \times 2^n} \times \left(\frac{1}{2}\right)^{22 \times 2^n - 7} \times \left(\frac{1}{6}\right)^{14 \times 2^n - 3}. \end{aligned}$$

In the next theorem, we compute the multiplicative sum connectivity index of $NS_3[n]$ dendrimer nanostar.

Theorem 6. The multiplicative sum connectivity index of $NS_2[n]$ dendrimer nanostar is

$$XII(NS_3[n]) = \left(\frac{1}{2}\right)^{2^{n+1}} \times \left(\frac{1}{2}\right)^{12 \times 2^n - 7} \times \left(\frac{1}{5}\right)^{14 \times 2^n - 3} \times \left(\frac{1}{6}\right)^{3 \times 2^n}.$$

Proof: We have $XII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$.

By using Table 3, we have

$$\begin{aligned} XII(NS_3[n]) &= \left(\frac{1}{\sqrt{1+3}}\right)^{2^{n+1}} \times \left(\frac{1}{\sqrt{2+2}}\right)^{22 \times 2^n - 7} \times \left(\frac{1}{\sqrt{2+3}}\right)^{28 \times 2^n - 6} \times \left(\frac{1}{\sqrt{3+3}}\right)^{6 \times 2^n} \\ &= \left(\frac{1}{2}\right)^{2^{n+1}} \times \left(\frac{1}{2}\right)^{22 \times 2^n - 7} \times \left(\frac{1}{5}\right)^{14 \times 2^n - 3} \times \left(\frac{1}{6}\right)^{3 \times 2^n}. \end{aligned}$$

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