

Structural Analysis of Rectangular Orthotropic Slabs with Openings and Short Boundary Conditions Using Yield Line Theory

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ABSTRACT

An endeavour has been made to evaluate the two way rectangular orthotropic slab with interior Enclose opening with one short boundary intermittent slab using yield line theory. Keeping in view the basic principles of yield line theory, all possible yield line patterns are considered for the given configurations of the slab subjected to uniformly distributed load (udl). A computer program has been developed to solve the virtual work equations derived in this paper. Relevant tables and charts for given data and the governing admissible failure patterns of the slab for different sizes of openings are presented using the affine theorem. In this paper, the authors also present the transformation of orthotropic slab into an equivalent isotropic slab using the affine theorem. The analysis is carried out with aspect ratio of opening quite different to that the slab.

Keywords: aspect ratio, disposition (configuration) interior encloses opening, affinity theorem, orthotropic slab, uniformly distributed load, ultimate load, ultimate moment and virtual work equations

I. INTRODUCTION

Slab is one of the most important and complex structural components of a building. The limit state of analysis of solid slab has been taken up by Johansen using yield line method. Johansen has analysed most of the isotropic slab and presented a simple “affinity theorem” to solve orthogonal slabs. Modern building industries need different types of openings of slabs to carry service lines, plumbing lines etc. Analysis of simply supported isotropic slab with central opening is presented first time by Zaslavsky. He has presented some design charts also.

An orthogonal slab with interior corner opening for one short edges discontinuous condition has been chosen in this thesis work.

Slabs: Slabs are plate elements forming floors and roofs of buildings and carrying distributed loads primarily by flexure.

Yield line method: The yield line theory which is also a limit state is proposed by Ingerslev and then greatly extended by Johansen.

When solid panels (supported on four sides) are subjected to a uniformly distributed load, they will deform with significant curvature in two orthogonal directions and hence must be analyzed as two-way slabs; in such a case, reinforcement can be provided for calculated moments in two directions.

Yield lines or rupture lines or fractured lines: A slab is subjected to gradually increasing uniformly distributed load (udl), the tensile reinforcement starts yielding at a certain stage (loading), giving rise to cracks along the line of maximum moment which are known as “yield lines or rupture lines or fractured lines”.

Yield line analysis: The yield line analysis is a powerful tool to analyse under reinforced solid slabs to obtain the failure moments in the slabs of various shapes supported on various boundary conditions. To predict the failure patterns of under reinforced slabs require considerable skill and experience. Once a failure pattern is predicted, then it can be analysed by using virtual work principle as suggested by Johansen. To obtain the correct mechanism of failure pattern, one has to try number of geometrically admissible collapse mechanisms by using method of virtual work. This method has been used by many researchers in analysing various slabs supported on different boundary conditions. In order to confirm the critical failure pattern thus obtained using virtual work principle, one can use segmental equilibrium method.

II. METHODOLOGY

The method of determining collapse loads based on principle of virtual work has proved to be a powerful tool for a structural engineer, despite it gives an upper bound value. The work equations are formed by equating the energy absorbed by yield lines and the work done by the external load of the orthogonal rectangular slab with

interior corner openings where a small displacement is given to the slab. The various parameters considered are aspect ratio of slab (r), the coefficient of orthotropy (μ), orthogonal moment coefficients ($k_x^1, k_y^1, I_1, I_2, I_3, I_4$) and sum of orthogonal moment coefficients (Σk), the coefficients which determine the size of opening (α, β) and edge distance of opening (r_5, r_6) of slab.

Since there are many geometrically admissible collapse mechanisms for a given slab with openings subjected to udl, one has to find the correct mechanism which gives the least load carrying capacity of slab using computer program similar to one used by Islam³. The work equation is given by

$$\sum \left[\int_A W \Delta dA \right] = \sum \left[\theta \int_s m_\theta ds \right] \text{Where,}$$

W = ultimate uniformly distribute load/unit area of the slab

Δ = deflection on the element

dA = unit area (dx)*(dy)

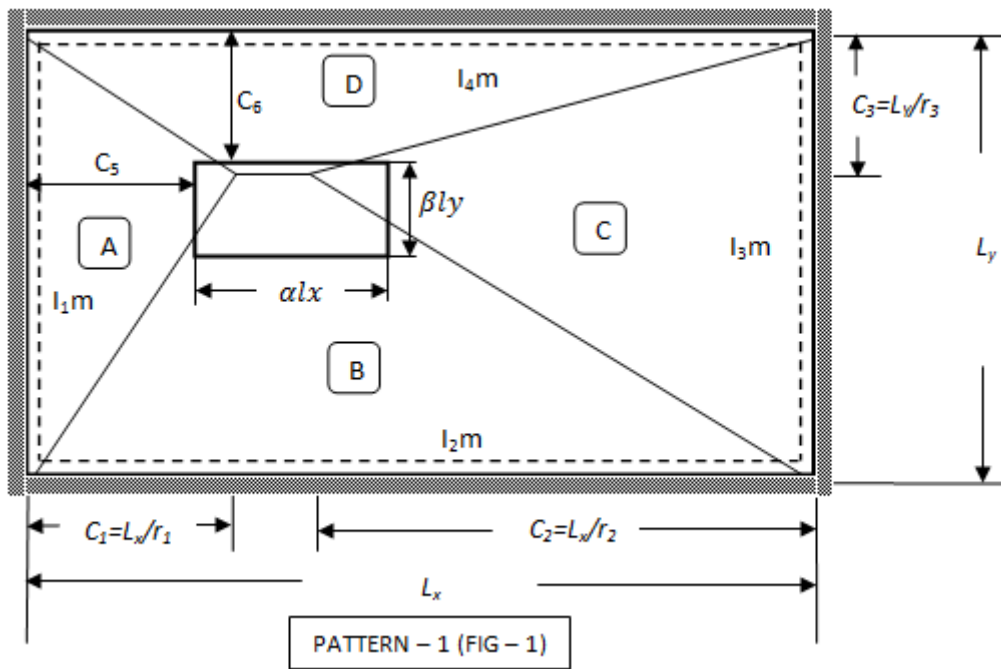
m = normal moment per unit length at a point in yield line.

θ = total rotation of yield line.

ds = short length of yield line at that point.

III. ANALYSIS OF COLLAPSE MECHANISMS

Formulation of virtual work equation: There are several possible yield line patterns associated with different edge conditions of the slab. For the OLC conditions of slab, the possible admissible failure yield line patterns are twenty, for the OLD edge conditions of slab, the possible admissible failure yield line patterns are twenty. These admissible failure yield line patterns are obtained basing on the principle of Johansen k.w.



Thirty four possible failure patterns are predicted for One Short Sides Discontinuous Slab (OSD). Considering the failure Pattern-1 of a continuous slab & let δ be the virtual displacement. Three unknown dimensions C_1, C_2 & C_3 are necessary to define the yield line propagation completely.

The external work done by segment A:-

$$\begin{aligned} &= \left[\frac{1}{2} C_6 X_1^2 \frac{W}{3C_1} + Y_1 C_5^2 \frac{W}{2C_1} + \frac{1}{2} Y_2 C_5^2 \frac{W}{3C_1} \right] \\ &= W L_x L_y \left[\frac{r_3^2 r_6^3}{6r_1} + \left[1 - \frac{r_1 r_5 (r_3 - 1)}{r_3} - r_6 \right] \frac{r_5^2 r_1}{2} + \frac{r_1^2 r_5^3 (r_3 - 1)}{6r_3} \right] \end{aligned}$$

Affine Theorem

Orthogonal moment coefficients adopted as per affine theory to generate tables and charts in this thesis work.

μ	0.25		0.5		0.667		1.0		1.5		2.0		4.0	
$\sum K$	3	4	3	4	3	4	3	4	3	4	3	4	3	4
K'_X	0.3	0.4	0.5	0.67	0.6	0.8	0.75	1.0	0.9	1.2	1.0	1.33	1.2	1.6
K'_Y	1.2	1.6	1.0	1.33	0.9	1.2	0.75	1.0	0.6	0.8	0.5	0.67	0.3	0.4
$I_1 \text{ or } I_3$	0.3	0.4	0.5	0.67	0.6	0.8	0.75	1.0	0.9	1.2	1.0	1.33	1.2	1.6
$I_2 \text{ or } I_4$	1.2	1.6	1.0	1.33	0.9	1.2	0.75	1.0	0.6	0.8	0.5	0.67	0.3	0.4

Designer's choice

Orthogonal moment coefficients adopted as per a designers choice to generate tables and charts in this thesis work.

μ	0.25		0.5		0.667		1.0		1.5		2.0		4.0	
$\sum K$	3	4	3	4	3	4	3	4	3	4	3	4	3	4
K'_X	0.3	0.4	0.3	0.33	0.5	0.6	0.8	0.5	0.5	0.9	0.7	1.0	0.8	1.0
K'_Y	1.0	1.2	0.8	1.0	0.6	1.0	0.6	0.6	0.5	0.5	0.5	0.5	0.3	0.3
$I_1 \text{ or } I_3$	0.3	0.4	0.7	1.0	0.7	1.0	0.7	1.5	1.3	1.5	1.0	1.67	1.6	2.2
$I_2 \text{ or } I_4$	1.4	2.0	1.2	1.67	1.2	1.4	0.9	1.4	0.7	1.1	0.5	0.83	0.3	0.5

Reverse of designer's choice

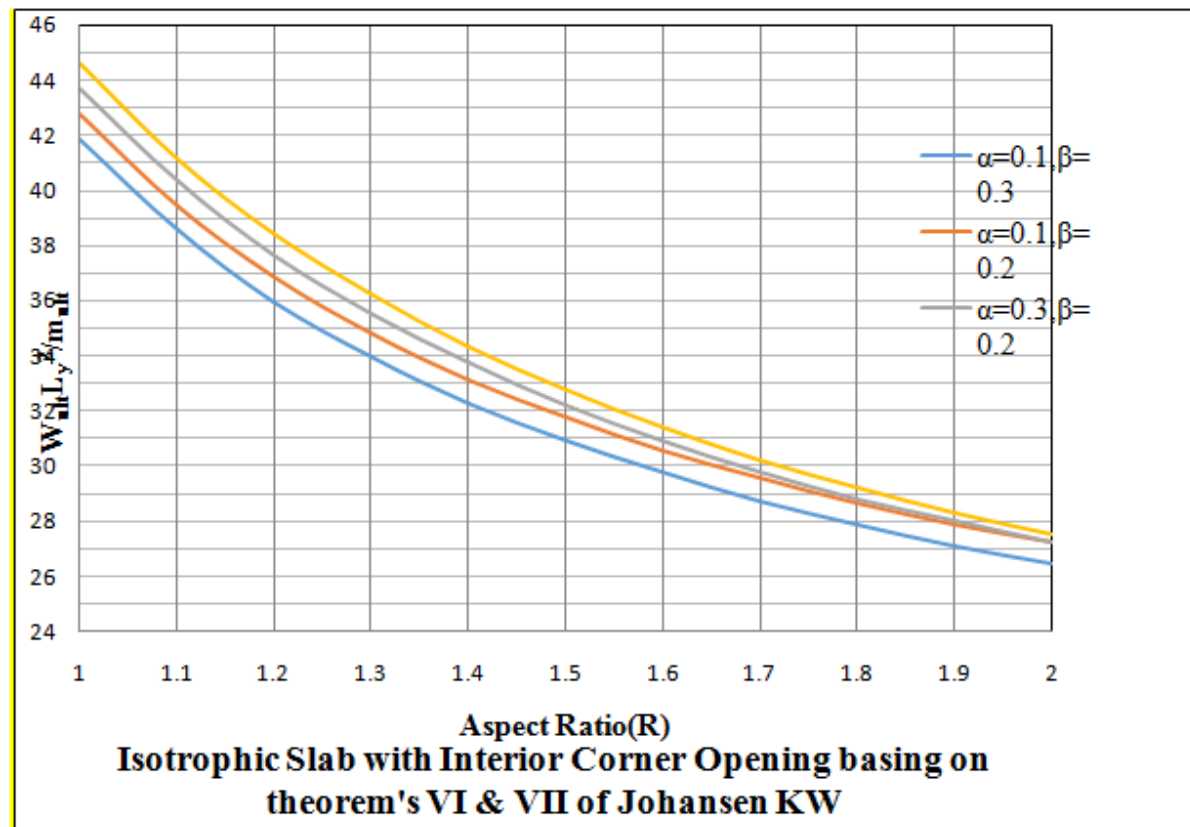
Orthogonal moment coefficients adopted to generate tables and charts in this thesis work by reversing the moment Coefficients given in the previous table

μ	0.25		0.5		0.667		1.0		1.5		2.0		4.0	
$\sum K$	3	4	3	4	3	4	3	4	3	4	3	4	3	4
K'_X	0.3	0.4	0.7	1.0	0.7	1.0	0.7	1.5	1.3	1.5	1.3	1.67	1.6	2.2
K'_Y	1.4	2.0	1.2	1.67	1.2	1.4	0.9	1.4	0.7	1.1	0.5	0.83	0.3	0.5
$I_1 \text{ or } I_3$	0.3	0.4	0.3	0.33	0.5	0.6	0.8	0.5	0.5	0.9	0.7	1.0	0.8	1.0
$I_2 \text{ or } I_4$	1.0	1.2	0.8	1.0	0.6	1.0	0.6	0.6	0.5	0.5	0.5	0.5	0.3	0.3

Minimization of virtual work: The value $W_{ult}Ly^2/m_{ult}$ of these equations consist of unknown non dimensional parameters r_1, r_2, r_3 and r_4 which define the positions of the yield lines. A computer program has been developed for various values of the non-dimensional parameters r_1, r_2, r_3 and r_4 within their allowable ranges in order to find the minimum value of $W_{ult}Ly^2/m_{ult}$ for the yield line failure pattern considered. In this computer program the values of r_1, r_2, r_3 were varied at increments of 0.001. Using the above equations, one can develop useful charts basing on orthogonality which may be used either for design or analysis in general.

IV. CONCLUSIONS

1. By using affine theorem for one short boundary intermittent slabs for $\alpha=\beta$ for different μ values charts are prepared.
2. A computer program is developed to evaluate all the Thirty four failure patterns .
3. Thirty four failure patterns are predicted to evaluate the strength of the slab with Inner section opening subjected to uniformly distributed load.
4. Few numerical examples are presented based on theorem of VI and VII of Johansen for orthotropic slabs with unequal openings A computer program is developed to evaluate all the Thirty four failure patterns
5. Isotropic charts are developed to various opening sizes in the slab for Affine Transformations
6. It is observed from the charts that the strength of slab with opening (for $\alpha=\beta$) is increasing when compared to solid slab.
7. The results obtained for good no. of problems by yield line theory are verified with equilibrium method.



V. REFERENCES

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